SHEAR STRESS ESTIMATION WITH A BIVANE ANEMOMETER

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1. INTRODUCTION

Friction velocity (u^*) is a fundamental variable in many processes of interest in micrometeorology, yet there is no straightforward method available for its measurement. Drag plates (e.g. Bradley, et al., 1968) can provide a direct measurement of shear stress, τ ($=\rho \overline{w'u'} = \rho u^{*2}$), under certain circumstances, but they require considerable care in fabrication, installation, and operation, and can scarcely be considered for routine use. Within the surface sublayer, τ can be determined by eddy correlation of fluctuations in u,v, and w obtained from a 3-dimensional sonic anemometer, but such instruments are expensive and perhaps not sufficiently robust for long-term, unattended operation. The only remaining alternative has been wind profile measurement, with subsequent estimation of u^* via K-theory, but this requires measurement of windspeed and temperature at multiple heights, empirical stability correction, and estimation of both roughness length and zero-plane displacement. A number of years ago, the suggestion was made that shear stress information might be extracted from mean anemometer bivane angles (Chimonas, 1968), but we are unaware of any subsequent research on the subject.

2. THEORY

The basis of the original argument was that even if the mean vertical velocity (w) is zero, correlation between w' and u' must result in a nonzero mean elevation angle, as shown in Fig.1 (after Chimonas, 1968). The instantaneous elevation angle of the bivane (assuming adequate frequency response of the vane) is given by:

$$\theta = -\arctan\left(\frac{\overline{W}+w'}{\overline{U}+u'}\right) \qquad [1]$$

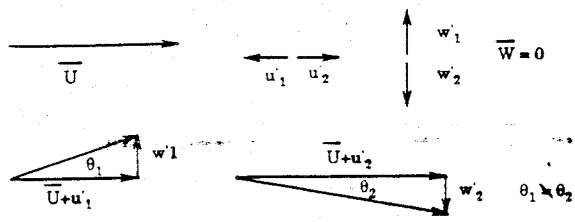

Figure 1. Fluctuations u'_1 and u'_2 about a mean horizontal velocity U correlated with fluctuations w'_1 and w'_2 about a mean vertical velocity of zero. The resulting angles, θ_1 and θ_2, are not of equal magnitude, hence the mean angle is non-zero.

A Taylor's series expansion of Eq. 1, up to and including 3rd order terms, followed by Reynolds' averaging, subject to the constraint that $\overline{w} > 0$, yields:

$$\overline{\theta} = 2\frac{\overline{w'u'}}{\overline{U}^2} - 2\frac{\overline{u'^2w'}}{\overline{U}^3} + \frac{\overline{u'^3w'}}{\overline{U}^4} - \frac{\overline{u'w'^3}}{\overline{U}^4} + \frac{\overline{u'^2w'^3}}{\overline{U}^5} \qquad [2]$$

Chimonas (1968) presented a somewhat different expression; most notably the factor of 2 was absent from the first term, for unknown reasons. The relative importance of the various terms in Eq. 2 is not immediately obvious. Intuitively, one expects that the higher order terms in Eq. 2 will diminish in importance with increasing U. If the four rightmost terms are arbitrarily small relative to the first term, or more generally if their algebraic sum is sufficiently small to allow the following approximation,

$$\bar{\theta} = 2\frac{\overline{w'u'}}{\bar{U}^2} = 2\frac{u_*^2}{\bar{U}^2} \qquad [3a]$$

or equivalently,

$$u_* = \bar{U}\left\{\frac{\theta}{2}\right\}^{0.5} \qquad [3b]$$

then a bivane anemometer should provide a straightforward means for estimating shear stress and friction velocity, provided it can respond accurately to all frequencies contributing significantly to momentum transfer. There are several questions which must be resolved, among them: what are the magnitudes of the various terms in Eq.2, and what are the possible sources of error in the measurement.

3. RESULTS

3.1 Components of the Mean Elevation Angle

This question was addressed by extracting the component terms of Eq. 2 from the output of a 3-dimensional sonic anemometer (Applied Technologies, Niwot, CO, USA), installed at 2 m above a soybean canopy. This instrument has a 10 cm pathlength and a fixed sampling rate of 10 Hz. An averaging interval of 20 min was used, and coordinate rotation was performed prior to computation of the various means. The results are summarized in Figure 2, which

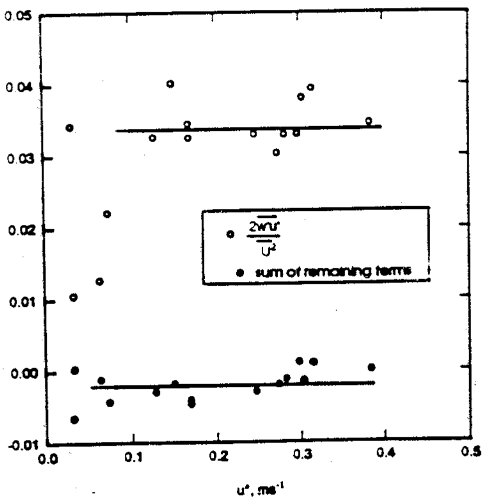

Figure 2. Data computed from 20 minute means of 3-dimensional sonic anemometer signals at 2 m above a soybean canopy. Solid line through the open circles is the mean bivane angle for the period.

shows the first term ($2\overline{w'u'}/\bar{U}^2$) as open circles and the algebraic sum of the remaining terms in Eq.2 as closed circles, plotted against u_*. The data suggest that the approximation given by Eq. 3 is supportable, except at very low friction velocities.

3.2 Frequency Response

Frequency response is of primary concern in this application, the more so in our case, since we hope to use the instrument for shear stress measurement at the field scale in locations where fetch constraints dictate that flux measurements be made within 2-3 m of the surface. A bivane is a 2nd order system, for which the characteristics of interest are the damping ratio (ζ) and natural wavelength (λ) (MacCready and Jex, 1964a). MacCready and Jex (1964b) addressed the question of vane frequency response with respect to measuring the variance of u. They derived a solution for vane output to a forcing function whose normalized spectrum approximates that of u in the inertial subrange, falling as $f^{-5/3}$, where f is a non-dimensional frequency (f=nz/U). They showed that the vane amplifies the signal in the region near the natural frequency of the vane due to overshoot, while underestimating at higher frequencies, with an overall relative error in the measured spectrum integral that depends on the bivane response properties. For the vane that we used (Gill bivane,R.M. Young Co., λ=4.8 m,ζ = 0.64) and the conditions under which we used it, an analysis following MacCready and Jex (1964) projects underestimates of 10-20 % in measuring the variance of u or w, with the largest errors under stable conditions. However, our interest is not in the variance in one dimension, but rather in the mean elevation angle, which presumably should follow the uw cospectrum. Kaimal, et al. (1972) indicate that the uw cospectrum peaks over a frequency range similar to that of the velocity spectra but falls off more rapidly in the inertial subrange, with a slope of -7/3 rather than -5/3. Thus the errors associated with diminshed response at high frequencies should be less serious for calculating mean elevation angles than they are for computing variances.

A bivane specifically designed for measuring mean elevation angle could further minimize frequency response concerns. MacCready and Jex (1964) describe general considerations in vane design; it seems that the most productive step to improve the high frequency response would be to reduce the size, and hence the natural wavelength. On a smaller bivane it would be difficult to retain a propeller anemometer on the front while still maintaining an acceptable damping ratio, but for our purpose the anemometer need not be an integral part of the vane, since $\bar{U}$ can be obtained from a separate anemometer.

3.3 Alignment Errors

It is implicit in the derivation that the bivane elevation angle is referenced to a plane that is normal to the momentum flux. When such is not the case, there is an offset error which complicates analysis. If θ is the measured elevation angle and α is the offset error, then Eq. 3 becomes

$$\overline{\theta - \alpha} = 2\frac{u*^2}{\bar{U}^2} \quad [4]$$

Algebraic manipulation of Eq.4 yields

$$\bar{U}\bar{\theta}^{0.5} = \{2u*^2 - \bar{\alpha}\bar{U}^2\}^{0.5} \quad [5]$$

We introduce a drag coefficient $C_d = 2\frac{\bar{U}^2}{u*^2\phi_m{}^2}$, where ϕ_m is the universal stability function. This allows the following substitution,

$$\bar{U}^2 = 2\frac{u*^2\phi_m{}^2}{C_d} \quad [6]$$

and Eq. 5 can be rewritten as

$$u* = \beta\,\bar{U}\left\{\frac{\bar{\theta}}{2}\right\}^{0.5} \quad [7]$$

where

$$\beta = \left\{1 - \frac{\bar{\alpha}\phi_m{}^2}{C_d}\right\}^{-0.5} \quad [8]$$

There is little utility to eq. 7 since there is no straightforward, independent means for evaluating β. Practical usage of the bivane for estimation of friction velocity requires that α be sufficiently small relative to Cd that β approaches unity, in which case Eq. 3b can be used and mean values of windspeed and elevation angle are sufficient for estimation of u*. Usage of Eq.3b in the presence of an alignment error will introduce a relative error in friction velocity estimation of $|1-\beta|$. If relative errors in estimation of u* are to be kept within 10%, then α must be no more than 18% of C_d.

3.4 Comparison with wind profile estimates

Field data were collected from a farm field on the University of Minnesota Agricultural Experiment Station at Rosemount, MN above a bare field. Elevation angle, azimuth angle, and mean windspeed were obtained from the previously mentioned Gill bivane anemometer (R.M. Young, Traverse City, MI, USA). Wind profile data included windspeed at 4 heights from cup anemometers (Gill microvane, R.M. Young Co.), differential temperature from Cu-Cn fine wire thermocouples, and temperature and humidity at bivane height from a Vaisala temperature/humidity sensor (Campbell Scientific). A 1-D sonic anemometer (CA21, Campbell Scientific) was used to obtain sensible heat flux. The bivane and the sonic anemometer were sampled at 10 Hz, while the profile instruments were sampled at 1 Hz. Means were computed at 20 minute intervals.

A scatter plot of friction velocity estimates by the bivane, using Eq. 3b, against stability-corrected wind profile estimates (Fig.3) produces a slope that is approximately 40% too large, indicating a non-negligible α nearly 50% as large as C_d. There is some evidence as well of an increasing tendency toward overestimation at high windspeeds, suggesting that vane frequency response may be a factor.

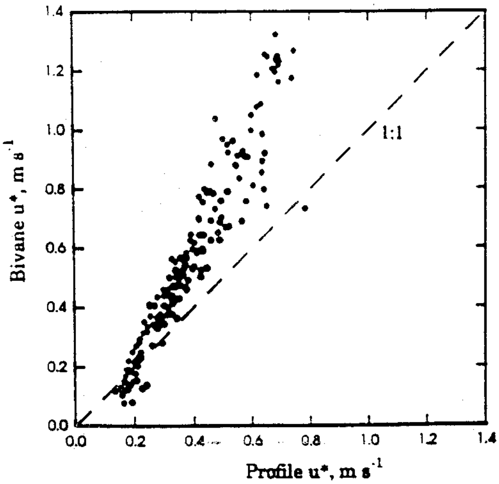

Figure 3. Scatter plot of friction velocity estimates from the bivane (using Eq. 3b) versus wind profile estimates, over a bare soil surface.

3.5 Comparison with eddy correlation estimates

In a subsequent trial over a soybean canopy, shear stress was measured with a 3-dimensional sonic anemometer 2 m above the crop, while elevation angle and windspeed were obtained from a nearby Gill bivane anemometer at the same height. The sonic anemometer was sampled at 10 Hz, while the bivane was sampled at 1 Hz. Subsequent spectral analysis of 10 Hz bivane data indicated that the 1 Hz sampling rate may have been too slow during a few intervals of high windspeed, but for most of the measurement period this was not a problem. Figure 4 is a scatter plot of the bivane u* estimates using Eq. 3b versus the eddy correlation estimates. In this case, the slope is quite close to unity, suggesing that α/C_d was much smaller than in the previous trial. This was probably due primarily to the rougher surface, and hence higher C_d. However, this measurement period did not include any periods of high u* (>0.6 m s^{-1}) comparable to those experienced during the earlier trial, and in that sense is a less rigorous test.

It may be difficult to obtain shear stress from bivane data over relatively smooth surfaces, where even small offset errors are significantly large relative to C_d. It is possible to increase C_d somewhat by measuring closer to the surface, but this imposes more stringent frequency response demands on the bivane. A final point worth noting is that Eq. 3b indicates that the true mean elevation angle during neutral

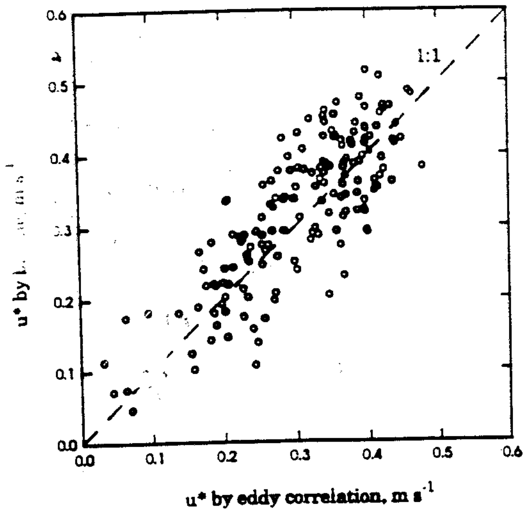

Figure 4. Scatter plot of bivane estimates of friction velocity, using Eq.3b, against eddy correlation estimates over a soybean canopy.

stability is equal to C_d. Thus tabled values of C_d for given surfaces, if available, might provide a means for estimating the presence and magnitude of elevation errors in bivane data. As a corollary, a bivane with negligible offset error could provide information about the form of the ϕ_m function, since the mean angle during non-neutral stability should equal C_d/ϕ_m.

4. CONCLUSIONS

A bivane anemometer may provide a straightforward, inexpensive alternative to existing approaches for momentum flux measurement, if elevation offset errors are small relative to the surface drag coefficient. This is not a trivial limitation, since such errors are difficult to eliminate or estimate. The resulting errors in friction velocity estimation scale with the square root of the ratio α/C_d, and hence are less serious over rough surfaces. From theoretical considerations the mean bivane angle, θ, is shown to be approximately equal to the drag coefficient under neutral conditions, and measurement of θ as a function of z/L may provide an additional means for confirming the functional form of empirical stability corrections. Additional research under a broader range of conditions, and with bivanes specifically designed for the purpose, should provide a better picture of the possibilities and limitations of the method.

5. REFERENCES

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