Measuring Soil Matric Potential *in situ* by Sensing Heat Dissipation within a Porous Body: I. Theory and Sensor Construction[1]

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ABSTRACT

A matric potential sensor is described that measures heat dissipation to sense the water content of a porous block in equilibrium with soil. It consists of a P-N junction diode that is surrounded by a heating coil and embedded in a porous medium.

A theoretical model is developed that describes the sensor in terms of its limiting parameters. Geometry, size, rate of heat input, and duration of heating are chosen to obtain a certain degree of accuracy and to make the measurements independent of the surrounding soil.

Construction, calibration, measurement, and temperature correction techniques are described. Since the electrical output of the sensor is linear with temperature, a single calibration curve is adequate at all temperatures within the accuracy reported.

By varying the composition of the porous body, the range and sensitivity of the sensor can be altered.

Additional Key Words for Indexing: model for soil matric potential sensor.

KNOWLEDGE of the matric potential of soil water over the range important in agriculture is essential for irrigation management. Several methods for determining matric potential of soil water are being used, but none of them is completely satisfactory. Most of the present methods that are applicable throughout the range important in agriculture are based on measurement of properties related to the water content of some porous medium whose matric potential is the same as that of the surrounding soil.

In a gypsum block, the water content is measured by the electrical resistance between two imbedded electrodes. Several problems are usually encountered when using gypsum blocks; first, salinity affects electrical conductivity independently of water content; second, the gypsum used in attempting to mask variations in soil salinity eventually dissolves, resulting in an unstable matrix for the sensor; third, contact resistance between the porous body and the soil can restrict the exchange of water between the block and the soil; and fourth, the hysteresis in the water content-matric potential relation of the porous body can cause errors in the interpretation of the data depending on whether the measurements are taken during a drying or wetting cycle. Elimination of the first problem requires a measurement of the water content of the porous body that is independent of salinity. This, in turn, would eliminate the need for the buffer material so that a stable matrix could be used. Contact resistance between the block and the soil can be minimized by packing the dry soil carefully around the sensor and then irrigating. Generally, the status of soil water is only of interest during drying conditions, since after a rain or an irrigation, the soil water content changes rapidly as the wetting front passes. Thus, hysteresis is of little concern and only a desorption calibration curve is needed.

The rate of heat dissipation in a porous medium of low heat conductivity has been shown to be sensitive to water content. Basically, it depends on the fact that air is a good thermal insulator with respect to water, so that as water is replaced by air, the remaining water films become thinner and the path length for heat conduction increases. This requires a larger temperature gradient to dissipate a given quantity of heat. The water content of a porous material can, therefore, be measured by supplying a heat source at a point centered within the block and measuring the temperature rise at that point.

Previous studies (Shaw and Baver, 1939; Bloodworth and Page, 1957; DeJager and Charles-Edwards, 1969) have demonstrated that heat dissipation in a porous medium can be used as an index of the amount of water present. Shaw and Baver (1939) inserted the temperature sensor-heater units directly into the soil. This required a separate calibration for each soil. Bloodworth and Page (1957) used a nonlinear temperature sensor (thermistor) so that either a separate calibration or a correction curve was required for use at a different temperature. In a personal communication, L. A. Richards indicated that the calibration procedure of an Electro-thermal Unit (US Patent #2,718,141, Sept. 20, 1955) had to be repeated periodically because of instability of the matrix material.

This paper presents a theoretical analysis and construction details for a matric potential sensor designed to eliminate some of these problems.

THEORY

The rate of heat dissipation in a porous body depends on the specific heat C, the thermal conductivity K, and the density ρ, of the material. For a homogeneous isotropic material whose thermal conductivity is not dependent on temperature, a useful relation known as the diffusivity, κ, is defined as $\kappa = K/\rho C$. In a porous material, the thermal conductivity depends on the conductivity of the matrix itself, the proportion of the material that is pore space, and the proportion of the pore space that is filled with water. K increases exponentially with water content. The specific heat increases almost linearly with water content as the air in the pore is displaced by water. The net results shown in Table 1 indicate that both the thermal conductivities and the thermal diffusivities for four different porous materials selected for possible use in the construction of the sensors increased with water content. These data were obtained by embedding a heater and temperature sensor within a block of each material as described in Appendix 2, heating at a constant rate, and determining K and κ from equation

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Table 1—Thermal conductivity, diffusivity,* and percent pore space for porous materials considered for matric potential sensors

Material used	Thermal conductivity air dry	Thermal conductivity water saturated	Thermal diffusivity air dry	Thermal diffusivity water saturated	Ratio K_{wet}/K_{dry}	Ratio $\kappa_{wet}/\kappa_{dry}$	Pore space
	cal/sec cm C		cm²/sec				%
Ceramic†	0.00114	0.00184	0.00118	0.00383	1.61	3.25	17.8
Refired ceramic‡	0.00087	0.00161	0.00131	0.00619	1.85	4.73	25.6
Castone and ground ceramic§	0.00121	0.00274	0.00068	0.00333	2.26	4.90	28.4
Gypsum¶	0.00076	0.00192	0.00101	0.00510	2.53	5.05	44.5
Water		0.00144**		0.00144**			
Air	0.000058**		0.187**				

* Conductivity and diffusivity calculated from equation [1], Appendix 2.
† Butcher 3,000 ceramic slip.
‡ Ground ceramic cups and Butcher 3,000 ceramic slip fixed at 950C.
§ White Castone and ground tensiometer cups (1 bar).
¶ Gray Hydrocal B-11 Gypsum.
** Carslaw and Jaeger (1960).

[1], Appendix 2. As expected, the greater the pore space (determined by the difference between the wet and dry weights), the less the conductivity was for each material in the dry state. If the absolute values of the conductivity are correct, the matrix materials had a conductivity greater than that of water as is shown by the fact that their saturated conductivities are all greater than that of water. The ratio of the conductivity and diffusivity of a porous material when dry to that when it is wet gives an indication of the precision with which its water content can be measured by the thermal method. Ideally this ratio should be as great as possible. Gypsum has the highest ratio in Table 1, primarily because of its large pore space percentage. Because of the lower conductivity of its matrix, ceramic would show a greater ratio of dry to wet thermal conductivity if its pore space percentage were as great as that of gypsum.

Equally important is the distribution of pore space. As will be shown below, although gypsum has a large pore space percentage, most of its pores are in the larger sizes that empty at high matric potential (low suction). The pore size distribution should be such that the block is not completely desaturated when the lower matric potential of −15 bars is reached.

A theoretical model is helpful in finding how variations in geometry and the rate and duration of heat input affect the sensitivity and accuracy of a sensor. Our model consists of two concentric spheres, the inner one containing a temperature-sensing device and a heater, and the outer one a porous body. We assume that the region $0 < r < a$, the inner sphere, is a perfect conductor; the outer sphere, the porous medium, occupies the region $a < r < b$. (See Appendix 1 for definition of terms.) We further assume perfect contact between the spheres at r = a, and that the initial temperature of the whole system is zero. Now if heat is applied in the sphere $r = a$ at the rate Q per unit time while the boundary $r = b$ is maintained at zero, Carslaw and Jaeger (1960) show that the temperature v, in the region $a \leq r \leq b$, is given by

$$v = \frac{Q}{4\pi K}\left(\frac{1}{r} - \frac{1}{b}\right) - \frac{Qk}{r\pi K a}\sum_{n=1}^{\infty} \frac{\sin \alpha_n (b-a) \sin \alpha_n (b-r)}{\alpha_n [2k(b-a)\alpha_n + 4a\alpha_n \sin^2 \alpha_n (b-a) - k \sin 2\alpha_n (b-a)]} \exp(-\kappa \alpha_n^2 t) \quad [1]$$

where $k = \dfrac{4\pi a^3 \rho C}{M_1 C_1}$

and α_n, $n = 1, 2 ---$, are the positive roots of

$$k\, a\alpha \cos \alpha(b-a) = (a^2\alpha^2 - k) \sin \alpha(b-a) \quad [2]$$

The first six positive roots of equation [2] were determined for values of b ranging from 1 to 5 cm by means of Mueller's iteration scheme of successive bisection and inverse parabolic interpolation (IBM, 1968). A Fortran program was then used to solve equation [1] numerically at the same values of b, using the roots from equation [2] and the values of ~~K and~~ κ calculated for gypsum in Table 1.

In general, it is desirable to keep the block as small as possible. However, for a sensor to measure matric potential uniquely regardless of the soil in which it is embedded, it must be large enough to contain the heat pulse. If it is not, the temperature at the center of the block after a heat pulse will depend not only upon the matric potential of the block, but also upon the thermal conductivity and the specific heat of the soil in which it is embedded. Data in Table 1 show a block's thermal conductivity and diffusivity are greatest when it is saturated with water. If we determine the minimum radius necessary to contain the heat pulse at saturation, it will be more than sufficient to contain the heat pulse at any lower water content.

Figure 1 shows the temperature at the center of a saturated gypsum block as a function of its radius for various heating periods and for a heating rate of 2 millical/sec. Note that for any given heating period there is a radius beyond which the temperature does not change significantly. Since the precision of our temperature sensor was about 2 millidegrees, any block whose temperature at its center after a heat pulse differed from that of an infinite block within this precision was considered sufficiently large to contain the heat pulse. The points in Fig. 1 represent the radius for each heating period that yields a temperature

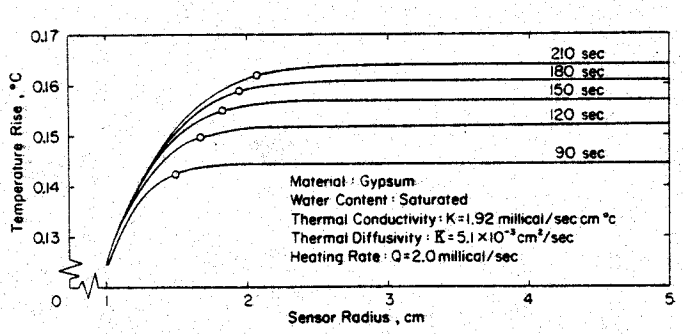

Fig. 1—Temperature increase of the diode as a function of sensor radius for various heating periods.

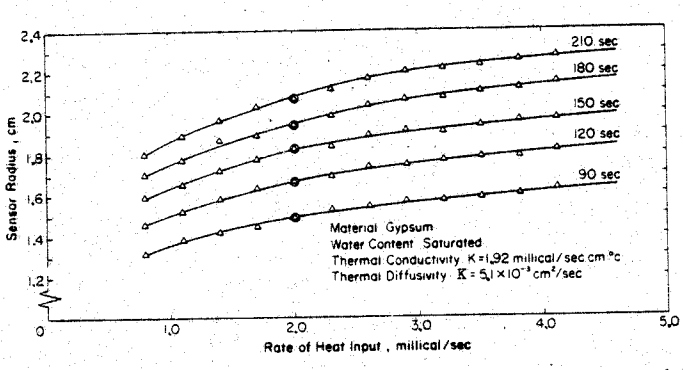

Fig. 2—Theoretical minimum sensor radius as a function of rate of heat input and heating period to insure that measurements are independent of surrounding medium.

within 2 millidegrees of that yielded by an infinitely large block. Similar families of curves were also constructed for heating rates of 0.8, 1.1, 1.4, 1.7, 2.0, 2.3, 2.6, 2.9, 3.2, 3.5, 3.8, 4.1, and 4.4 millical/sec.

Figure 2 was constructed by taking the points shown in Fig. 1 and similar curves and plotting them as a function of each rate of heat input. The resultant curves give the effect of the rate of heat input on the minimum radius necessary to contain the heat pulse for various heating periods. These curves form the basis for choosing the critical parameters in designing matric potential sensors. This particular set of lines is applicable only for a body of gypsum, but similar data could be obtained for other porous materials simply by substituting the appropriate thermal conductivity and diffusivity into equation [1]. Because gypsum had one of the highest saturated thermal conductivities and diffusivities, the radius given in Fig. 2 should be large enough to contain the heat pulse for any of the other materials listed in Table 1. Hence, by reference to Fig. 2 and Table 1, a sensor can be built that should measure matric potential uniquely regardless of the soil in which it is embedded, since a heat pulse can be applied in such a way as to be contained entirely within the porous body.

DESIGN AND CONSTRUCTION OF THE UNIT

An RCA 1N2326 Germanium P-N Junction Diode was chosen as the temperature sensor because its characteristics and availability made it practical at the early stage of design. (The citation of particular products or companies is for the convenience of the reader and does not imply any endorsement, guarantee, or preferential treatment of the US Dept. of Agriculture or its agents.) Actually, its size and geometry do not really satisfy the assumed condition of infinite conductivity of the inner sphere in the theory, but the deviations should not be serious. Smaller silicon junction diodes are now being tested for this purpose.

Sargent (1965) showed that the exponential dependence of the reverse saturation current of the diode on temperature and the forward current exponential dependence on junction voltage interact to give a linear functional relationship between junction voltage and temperature. If the difference in voltage before and after heating is used as the readout, this linear temperature relationship eliminates the need for sensor calibration at various temperatures provided that the matric potential dependence of the porous body on temperature does not exceed the accuracy of the measurement. This makes the unit practically independent of temperature in an environment free of rapid temperature fluctuations.

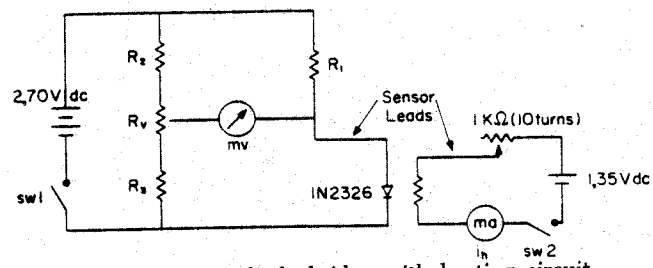

Fig. 3—Basic diode bridge with heating circuit.

The bridge circuit used for temperature measurement is shown in Fig. 3. If the battery voltage is kept constant and large with respect to the junction voltage of the diode, the diode forward current will be nearly constant and can be changed if necessary by varying the potentiometer, R_v. When the above conditions are met, the junction voltage is a linear function of temperature with a negative slope for temperatures ranging from −200 to +50C (Sargent, 1965; Barton, 1962; McNamara, 1962; and Tanner, C. B. *Personal communication, 1966, Diode and transistor thermometers, CIC Summer Biometeorology Instrumentation Course lecture notes, Univ. of Wisconsin*). The values of R_1, R_2, R_3, and R_v are chosen as 1*K*, 1.270 *K*, 45, and 50 ohms to provide optimum performance for a temperature range from 10 to 40C. The bridge is powered by two RM42R mercury cells giving a total emf of 2.70 volts. Phene et al. (1969) have shown that the bridge and diode can be used for more than a year with the same batteries without loss in stability.

Various materials such as Hydrocal gypsum, (Gray-Hydrocal B-11 Gypsum, obtained from US Gypsum Company) ceramic, mixtures of ground ceramics, and Castone (Castone is a casting material used mainly in dental work; it is manufactured by Ransom and Randolph Co., Toledo, Ohio) were tested to select a stable matrix for enclosing the sensing unit. At present, it seems that only the Castone and ceramic materials are feasible, with the ceramic offering the greatest stability.

For practical purposes, a cylindrical block was used. If our theoretical sphere of radius $r = b$ is contained within the cylindrical block, the heat flow problem will not be changed and the only factors that will differ will be the surface area of contact between the block and the soil and the amount of material used. If necessary, spherical geometry will be used eventually to minimize the equilibration time.

Figure 4 is a cross-sectional sketch and Fig. 5 is a photograph of the matric potential sensor and its internal parts. The sensor consists of the P-N junction diode (RCA 1N2326) on which 200 cm of Teflon-coated copper wire (type T-40 DTP, 40-gauge, Thermoelectric Co., Inc.) has been wrapped to serve as the external heater element. Diodes are selected to have the same characteristics so that the sensors will have the same temperature sensitivity (Sargent, 1965; Tanner, pers. comm. cited above). Spaghetti tubing and heat-shrinkable rings are then adapted to the leads and the diode case as shown in Fig. 4. The heat-shrinkable rings serve as retainers for the heater wire that is wrapped as evenly as possible. Spray urethane (Uralane 8267, Furane Plastics, Inc.) is applied in three thin coats with a 24-hr drying period between each coat to hold the heater wire in place and to protect it from corrosion. The Teflon insulation is scraped off both ends of the heater wire and the leads are soldered to them. The heater and diode leads are then held together with heat-shrinkable plastic tubing and another coating of spray epoxy is applied over the whole assembly. After curing is complete, the sensor is ready to be embedded into the porous block.

If either Castone mix or gypsum is used, the sensing unit is centered in an acrylic plastic mold and the mixture is poured slowly while tapping the mold to eliminate air bubbles. The unit is then placed in a desiccator and slightly evacuated for 5 min. The mixture crystalizes rapidly and can be taken out of

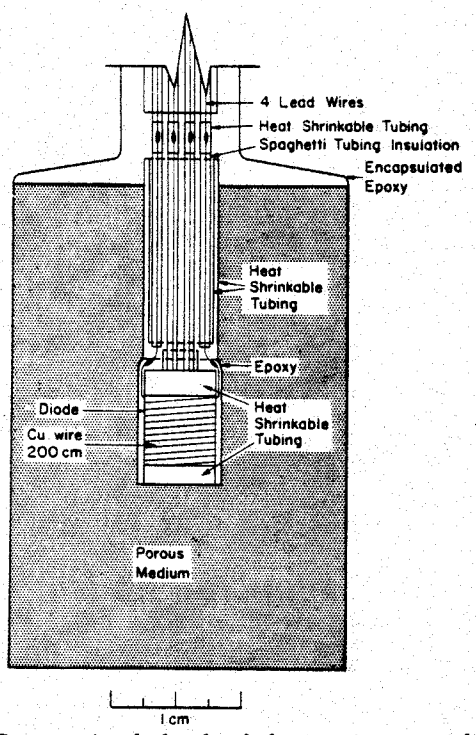

Fig. 4—Cross-sectional sketch of the matric potential sensor.

the mold to cure. If ceramic is used as the porous medium, the slip is poured into a plaster of Paris mold until it forms a solid cylindrical block. The block is allowed to cure in the mold until it separates easily. It may then be removed to dry at a faster rate. Ceramic blocks are air dried, then placed in an oven, and fired for 8 hr at a temperature greater than 900, but not exceeding 950C. After cooling, a cavity of dimension equal to the outside dimensions of the sensing unit is drilled into the center of the cylindrical block. Thermal conductive epoxy (EPO-TEK 410E, Epoxy Technology, Inc.) is used to establish contact between the sensing unit and the drilled cavity walls. After the epoxy has cured, the unfilled portion of the cavity is filled with a Castone ceramic mix and allowed to dry. At this stage, the leads are soldered to a shielded wire that accommodates four conductors, and the junctions are insulated with heat-shrinkable tubing. The top part of the unit is encapsulated in epoxy to give structural strength and to protect the area where the sensor leads enter the block.

TEST OF THE MODEL

At this point a simple test was used to determine whether or not the block size, the rate of heat input, and the heating period predicted from Fig. 2 were adequate to contain the heat pulse within the block. The test consisted of taking measurements with a water-saturated block in contact with air and in contact with mercury. When in contact with air, the block was enclosed in a large test tube to prevent evaporation. The difference between the thermal conductivities of air and mercury is far greater than that between any soils. Thus, if the temperature at the center of the block following a heat pulse was no different with the block in air than in mercury, we can be sure that textural differences in the soil will not affect the reading. Furthermore, because the theory assumed no contact resistance at the boundary $r = a$, we expected the restrictions imposed by the model to be slightly conservative.

A gypsum block of 1.6-cm radius was used with rates of heat input of 0.81, 1.44, and 3.25 millical/sec, applied for 150 sec each. Figure 2 predicts that for a radius of 1.6 cm and a heating cycle of 150 sec, the maximum rate of heat input

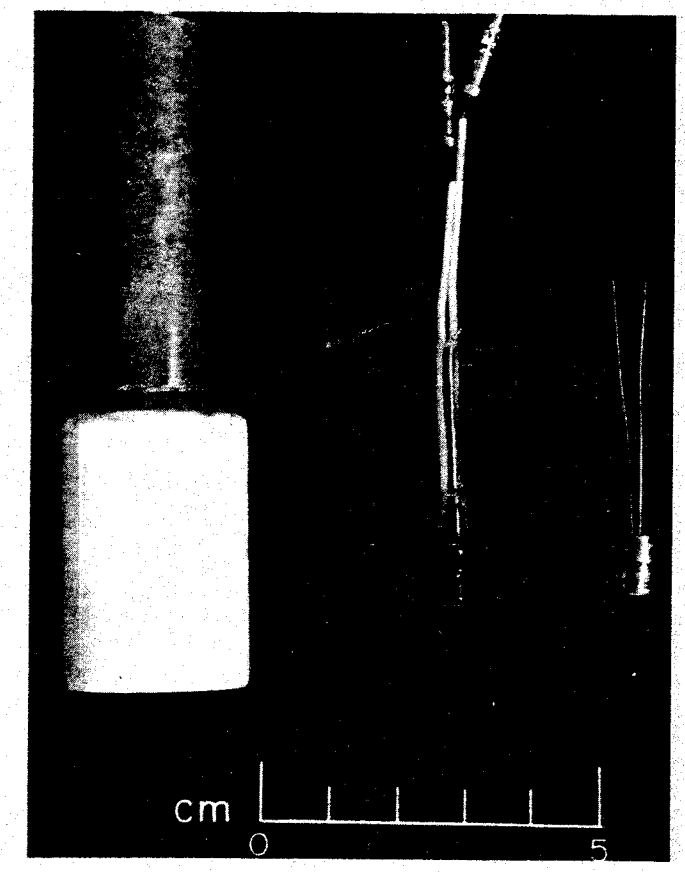

Fig. 5—The diode (*right*), the diode and heater (*center*), the matric potential sensor (*left*).

should not exceed 0.85 millical/sec. The results of the test showed that for rates of heat input of 3.25 and 1.44 millical/sec the mean of 10 temperature measurements at the center of the water-saturated block increased by 0.009 and 0.005C, respectively, when the block was moved from mercury to air at constant temperature. When the rate of heat input was decreased to 0.81 millical/sec, the difference of the mean was less than 0.001C and within the accuracy of the system. The precision of the sensing unit under ideal conditions is 0.002C. If this precision can be obtained when the block is moved from mercury to air, then the heat pulse is contained within the block. Accordingly, the results of the tests were adequate only for a rate of heat input less than that predicted by the theory. Therefore, it can be concluded that the model was accurate in predicting the rate of heat input to restrict the heat flow to the porous body.

CALIBRATION PROCEDURE

Matric potential measurements are obtained by taking two voltage readings of the bridge. The first one is taken before the heating cycle begins and the second is taken after the heating cycle. The difference of the two voltage measurements represents the change in temperature at the center of the sensor due to the heat applied. The magnitude of the difference varies depending on the water content of the porous block, which directly affects the rate of heat dissipation. The calibration of the voltage differences versus matric potential is accomplished at constant temperature in a pressure plate apparatus. Since the amount of surface area in contact with the soil affects the response rate of the sensor (Willis, W. O., and J. R. Hadley, 1959. Electrothermal unit for measuring soil suction. US Salinity Lab. Research Report no. 91), it is advantageous

to have as large a contact surface as possible between the sensor and the soil used for calibration. To do this, the sensors were placed under spring tension in dry soil within 15 bar bubbling pressure ceramic calibrating cups (Mayhugh, M. S., 1960. Electrothermal unit for measuring soil suction. II. Improved calibration procedure. US Salinity Lab. Research Report no. 94). The soil was then saturated. Readings were taken when nearly constant measurements were obtained and there was no water flow from the pressure plate outflow tube. The last 12 measurements taken over a 24-hour period were used to determine the mean and standard deviation of the measurement. The pressure on the plate was then increased. Calibration was carried out only for a desorption cycle and no attempt was made to study hysteresis. To avoid the effect of pressure on heat transfer in the gaseous phase within the sensor, the pressure should be released from the pressure apparatus (Willis and Hadley, 1959; Mayhugh, cited above) and the outflow tube evacuated and clamped. Releasing the pressure causes adiabatic cooling and water may flow. Hence, time must then be allowed for temperature to reach equilibrium.

EVALUATION

The sensitivity of the sensor was changed by using different porous bodies. The calibration curves shown in Fig. 6 are representative of the different ranges obtained. Although curve 1 (gypsum) shows the highest total response, gypsum's high solubility precludes its use for long experiments. The large sensitivity at the higher matric potentials permits readings with an accuracy of better than ± 0.1 bar between matric potentials of 0 and −1 bar. The calibration curve may be plotted as a straight line on semilog paper so that the results are easily interpreted. Curves 2 and 3 are obtained from a porous body prepared by grinding porous ceramic (broken tensiometer cups), mixing 2.1 parts of this with 1.7 parts of Castone and 1 part of water, and allowing the mixture to harden in a mold. Curves 4 and 5 are obtained from a similar mixture, except only 1.6 parts of ground ceramic were used. Curve 6 is obtained from a fired ceramic porous body. The ceramic provides a solid stable matrix whose characteristics will not change with time unless microorganisms accumulate on its surface and restrict water flow. The other advantage of the ceramic is its linear relationship between 0 and −6 bars matric potential. Experiments are being conducted to find a porous medium that gives a linear output between 0 and −10 bars matric potential.

The matric potential of porous bodies is dependent upon temperature. This dependence is also a function of the texture of the material. Richards and Weaver (1944) have shown that the change in water retention per degree C change in temperature at matric potentials of −0.5 and −15 bars increases with a texture change from coarse to fine but does not appear to be a linear function of the water retention of the various soils at any given temperature and tension. Gardner (1955) presents data which show that matric potential decreases at a rate of approximately 0.008 bar per degree C rise in temperature for matric potentials between 0 and −0.5 bar and for temperatures ranging from 0 to 50C. A personal communication from Dr. J. D. Oster indicates that for a calcium-saturated Gila clay a change of −0.01 bar/bar per degree C increase was measured. Figure 7 shows no temperature dependence of the matric potential measurements in the upper matric potential range for a Castone porous body within the accuracy of our measurement. Below −10 bars matric potential, the temperature dependence is significant because of the decrease in sensitivity of the sensor.

The accuracy of the measurement is strongly dependent upon the time rate of change of soil temperature. The range of matric potential from 0 to −10 bars corresponds to a change in diode temperature of 0.3C at the most, or 0.03C per bar if the relationship were linear.

To avoid the error caused by the change in soil temperature, the measurement can be taken when the soil temperature is constant. In the laboratory, temperature can be controlled so that the problem does not exist. In the field, the soil temperature varies sinusoidally with time over a 24-hr period. The amplitude of the wave is a func-

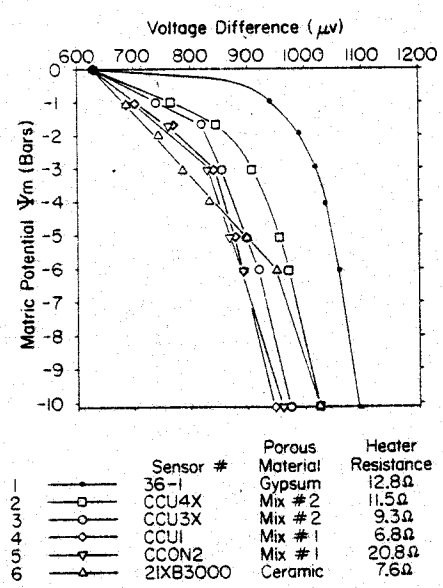

Fig. 6—Effect of different porous materials on sensitivity of the sensor.

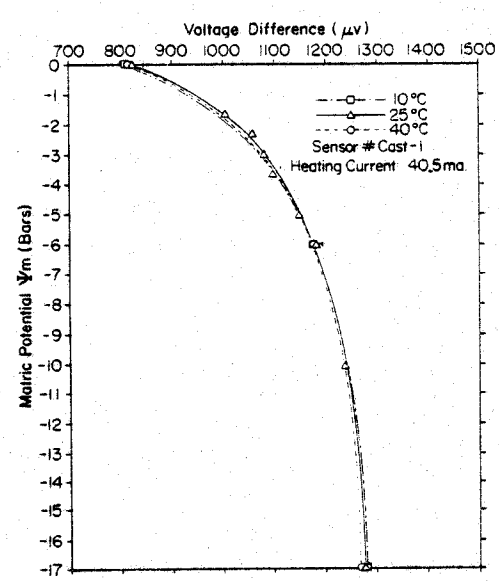

Fig. 7—Effect of temperature on sensor calibration.

tion of depth, soil, water content, crop cover, heat load, etc. The rate of temperature change is least at those times corresponding to the maximum or minimum in this wave. The temperature function can be determined accurately with the diode. Although the amplitude of the wave is not constant, the times at which its maximum and minimum occur do not drift much from day to day unless drastic climatic changes take place.

Another way to avoid the error caused by the soil temperature change could consist of replacing the resistor R_3 (Fig. 3) with a second diode encapsulated in a separate porous body of the same material. This diode would be subjected to the same thermal changes in the soil and would serve as a reference for the diode in the matric potential sensor. For small temperature changes, the effect of temperature on water flow can be neglected and the bridge voltage measured would be independent of temperature changes. This method requires careful matching of the diodes and a strict calibration of the two diodes in the bridge (Tanner, pers. comm. cited above). Theoretically, the heat flow problem would also be different in that the boundary conditions used for equation [1] are not constant.

CONCLUSIONS

The sensor could be modified to simplify its use and increase its sensitivity. Perhaps the most obvious simplification in its use would result from use of a new circuit in conjunction with a two-diode unit. This would make the sensor practically temperature independent. At present sensitivity is limited by the relatively high heat conductivity of the dry porous material. If a porous material with a stable matrix, an adequate pore size distribution, and low heat conductivity could be found, the sensitivity of the unit would improve greatly and the precision of the measurement would increase. Finally, the linearization of the sensor's output with matric potential would eliminate the tedious calibration at multiple potentials and would greatly simplify the conversion of data. We are trying to achieve some of these improvements.

Meanwhile, the advantages of the existing sensor are:

1) Variable range and sensitivity with the possibility of obtaining a linear relationship between 0 and −15 bars matric potential make the sensor flexible.

2) The calibration dependence on temperature for a range of matric potential between 0 and −10 bars is within the 0.002C precision of the sensor when constantan wire is used for the heating element, Castone is used for the porous body, and the temperature range is between 10 and 40C.

3) Matric potential sensors using ceramic or Castone porous bodies have been used over periods of months without showing appreciable changes in output characteristics when recalibrated.

4) Measurements are well suited for portable or automatic devices.

5) The measurement is independent of the salt content of the soil solution.

6) The matric potential sensor gives a record of the soil temperature simultaneously with the matric potential with no extra measurement needed.

7) Construction is relatively easy and the cost is moderate when compared to existing methods.

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APPENDIX 1

Definition of terms:

C = specific heat of block at a known water content (cal/g C).
C_1 = specific heat of temperature sensor (cal/g C).
H^{-1} = contact resistance in semi-infinite spherical case (dimensionless).
K = thermal conductivity of porous medium (cal/sec cm C).
M_1 = mass of temperature sensor (g).
Q = heat applied per unit time (cal/sec).
R_1, R_2, R_3 = bridge resistance (ohm).
R_v = bridge 10-turn precision potentiometer (ohm).
a = lower bound value of r, radius of diode sensing unit (cm).
b = upper bound value of r, radius of block (cm).
h = H/K.
k = $4\pi a^3 \rho C/M_1C_1$.
r = variable radius value (cm).
t = heating period (sec).
v = temperature of sensor at $a \leq r \leq b$ (C).
v_2 = temperature of sensor for large values of time in semi-infinite medium (C).
α_n = roots of equation [2] $n = 1, 2 --- 6$.
κ = diffusivity of porous medium (cm²/sec).
ρ = block density at some known water content (g/cm³).

APPENDIX 2

Determination of thermal conductivity and diffusivity (see Appendix 1 for definition of terms):

In the model, the thermal conductivity K, and the thermal diffusivity κ, of the porous body had to be determined for

the particular material used. These parameters are highly dependent upon pore space and pore size distribution so that the thermal conductivity and diffusivity of two sensors made out of the same material might not be the same. Equation 20, page 350 of Carslaw and Jaeger (1960) gives the solution of the heat equation used for determining the thermal conductivity and diffusivity of materials for an infinite spherical region. Satisfactory performance of the method is dependent upon good contact at the interface between the temperature sensor and the porous body. When adequate contact is achieved the conductance H is high and $h >> 1$.

For t greater than 120 sec, the first two terms of the equation may be used to obtain the temperature v_2

$$v_2 = \frac{Q}{4\pi Ka}\left(\frac{1 + ah}{ah} - \frac{a}{(\pi \kappa t)^{1/2}}\right). \quad [1]$$

The measured value v_2 varies almost linearly as a function $1/\sqrt{t}$. Since $ah >> 1$, the term $(1 + ah)/ah$ reduces to 1 and the line has an intercept $B = Q/(4\pi Ka)$ and a slope $A = -Q/(4\pi^{3/2}K\sqrt{\kappa})$. Since Q and a are known, the conductivity, K, can be calculated from the intercept. The diffusivity, κ, can then be obtained by substituting the calculated value of K into the equation for the slope. The calculated values of K and κ for the various materials used are shown in Table 1 for water-saturated and air-dry cases.